

Introduction to Template Meta-Programming

Lectures on Modern Scientific Programming
Wigner RCP
23-25 November 2015

... C++ ...

C++ is a complex language...





Languages inside C++

C++ has 4 inner languages (as far as we know):

- The „normal”, „traditional” value-level
- The type- or template-level
- The C [preprocessor](#)
- And another [obscure](#) level based on whether a function is defined already or not


Languages inside C++

C++ has 4 inner languages (as far as we know):

- The „normal”, „traditional” value-level  This is what we talked about up to now
- The type- or template-level  This is by accident a functional language
- The C preprocessor  This can be seen again as a generic language
- And another obscure level based on whether a function is defined already or not  This is... you better not want to know☺

Languages inside C++

C++ has 4 inner languages (as far as we know):

- The „normal”, „traditional” value-level
- The type- or template-level 
- The C preprocessor
- And another obscure level based on whether a function is defined already or not

Lets see this

C++ templates

What are templates exactly?

C++ templates

Templates are „code templates” for the compiler.

They have „placeholders” that can be filled in later.

This „fill of placeholders” is called *instantiation*.

After instantiation, it is processed (typechecked, compiled etc.) as if it was written by hand by the programmer.

C++ templates

Example:

This is a class template definition (despite it is a struct, never mind)

```
template<typename T>
struct Circle
{
    T r;
    T area() const { return r*r*3.14; }
};
```


C++ templates

Example:

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```
template<typename T>
struct Circle
{
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};
```

These are two class template instantiations:

```
Circle<float> c1;
Circle<double> c2;
```

C++ templates

Example: This is a class template definition (despite it is a struct, never mind)

```
template<typename T>
struct Circle
{
    T r;
    T area() const
    { return r*r*3.14; }
};
```

```
struct Circle
{
    float r;
    float area() const
    { return r*r*3.14; }
};
```

```
struct Circle
{
    double r;
    double area() const
    { return r*r*3.14; }
};
```

These are two class template instantiations:

```
Circle<float> c1;
Circle<double> c2;
```

These are equivalent to the above explicit structs.

Templates can be specialized:

Base template

```
template<typename T>
struct Circle
{
    T r;
    T area() const
    { return r*r*3.14; }
};
```

Specialization for `int`:

```
template<>
struct Circle<int>
{
    int r;
    double area() const
    { return (r*r)*3.14; }
};
```

C++ templates - SFINAE

Another example of specialization:

```
template<bool B, typename T = void>  
struct enable_if {};
```

```
template<class T>  
struct enable_if<true, T> { typedef T type; };
```

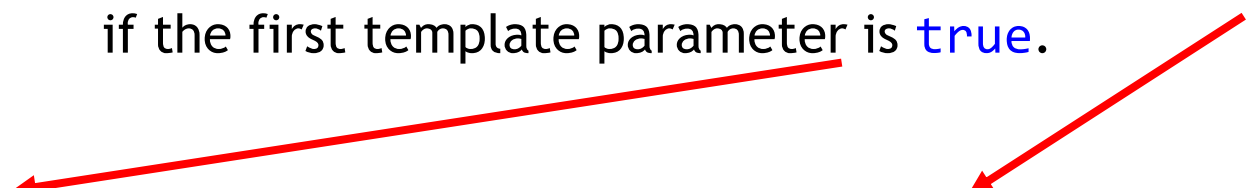
C++ templates - SFINAE

Another example of specialization:

```
template<bool B, typename T = void>  
struct enable_if {};
```

This struct only possess a member type called: `type`,
if the first template parameter is `true`.

```
template<class T>  
struct enable_if<true, T> { typedef T type; };
```



C++ templates - SFINAE

SFINAE

C++ templates - SFINAE

SFINAE - The worst acronym in history

SFINAE - Substitution Failure Is Not An Error

SFINAE - Substitution Failure Is Not An Error

When a template instantiation is being resolved, multiple possible cases should be inspected for a possible match.

If some of the matches turn out to be ill-formed it will not be a compile error, it will be silently excluded from the set of possibilities.

C++ templates - SFINAE

Consider:

```
template <typename T,  
         typename Checker = void>  
struct Matrix  
{...};
```

```
template <typename T>  
struct Matrix<T, typename enable_if<is_noncommutative<T>::value>::type>  
{...};
```


C++ templates - SFINAE

Consider:

```
template <typename T,  
         typename Checker = void>  
struct Matrix  
{...};
```

`is_noncommutative` should be a similar template like `enable_if`, but it contains a `bool` member called `value` if its `T` argument is from a non-commutative algebra. The `Matrix` then can be specialized for that case.

```
template <typename T>  
struct Matrix<T, typename enable_if<is_noncommutative<T>::value>::type>  
{...};
```



C++ templates - SFINAE

SFINAE

Functions can be templates too,
but this interferes with overload-resolution...

Prepare for surprises.

C++ templates - expression SFINAE

Expression SFINAE in C++11
Things get more gets simpler

C++ templates - expression SFINAE

```
template<typename T> struct Circle
{
    T r;
    T area() const { return r*r*3.14; }
};
```

```
struct SomeShape
{
    double area;
};
```

C++ templates - expression SFINAE

```
template<typename T> struct Circle
{
    T r;
    T area() const { return r*r*3.14; }
};
```

Here area is a function!

```
struct SomeShape
{
    double area;
};
```

Here area is a variable!

C++ templates - expression SFINAE

```
template<typename S>  
auto getArea( S shape )->decltype( shape.area() )  
{  
    return shape.area();  
}
```

```
template<typename S>  
auto getArea( S shape )->decltype( -shape.area )  
{  
    return shape.area;  
}
```


C++ templates - expression SFINAE

```
template<typename S>  
auto getArea( S shape )->decltype( shape.area() )  
{  
    return shape.area();  
}
```

`decltype` is an operator since C++11, that calculates the type of the expression inside!

```
template<typename S>  
auto getArea( S shape )->decltype( -shape.area )  
{  
    return shape.area;  
}
```

C++ templates - expression SFINAE


```
template<typename S>  
auto getArea( S shape )->decltype( shape.area() )  
{  
    return shape.area();  
}
```

This is only valid for types
having an area function



```
template<typename S>  
auto getArea( S shape )->decltype( -shape.area )  
{  
    return shape.area;  
}
```

This is only valid for types
having an area variable



C++ templates - expression SFINAE

```
template<typename S>  
auto getArea( S shape )->decltype( shape.area() )  
{  
    return shape.area();  
}
```

getArea(Circle<float>{1.0})
binds here, since Circle has an area() function

```
template<typename S>  
auto getArea( S shape )->decltype( shape.area )  
{  
    return shape.area;  
}
```

getArea(SomeShape{1.0})
binds here, since SomeShape has an area variable

C++ templates - expression SFINAE

With (expression) SFINAE we can specialize generic (functions) classes based on whether a given *operation* is valid on the argument types!

C++ templates - type_traits

The C++ Standard Library comes with many handy templates for checking properties of types.

C++ templates - TMP

Lets turn to template metaprogramming!

C++ templates - TMP

If the template level is a programming language...
What are the values?

The values are types:

```
struct x;
```

C++ templates - TMP

If the template level is a programming language...
What are the functions?

The functions are templates:

```
template<typename x> struct f;  
template<typename x, typename y> struct f2;  
template<int i> struct g;
```


C++ templates - TMP

If the template level is a programming language...
What are the functions?

Example functions:

```
template<typename x>  
struct id{ using result = x; };
```

```
template<int x> struct sq  
{  
    int value = x*x;  
};
```

The name of the member, where we make the “return value” of the function available is a matter of **convention!**

C++ templates - TMP

If the template level is a programming language...
What are the functions?

Example functions:

```
template<typename x>  
struct id{ using result = x; };
```

Equivalent to:

```
decltype(x) id(x){ return x; }
```

```
template<int x> struct sq  
{  
    int value = x*x;  
};
```

Equivalent to:

```
int sq(int x){ return x*x; }
```

C++ templates - TMP

If the template level is a programming language...
What are the functions?

Example functions:

```
template<typename x>  
struct id{ using result = x; };
```

```
template<int x> struct sq  
{  
    int value = x*x;  
};
```


In practice, non-type templates are not preferred, because they cannot be abstracted over.

Things get much simpler, if we raise these values to the type level

C++ templates - TMP

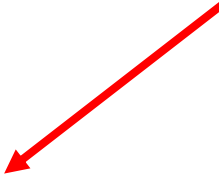
```
template<int x>
struct Int
{
    static const int result = x;
};
```

Type level integer value



```
template<typename x> struct sq{};
template<int i> struct sq<Int<i>>
{ using result = Int<i*i>; };
```

Specialization



C++ templates - TMP

Can't we create a generic implementation?

C++ templates - TMP

Can't we create a generic implementation?

Idea: `decltype` + operators

C++ templates - TMP

```
template<int x>  
struct Int  
{  
    static const int result = x;  
};
```

← Type level integer value

```
template<int i1, int i2>  
auto operator*( Int<i1>, Int<i2> )  
{  
    return Int<i1*i2>();  
}
```

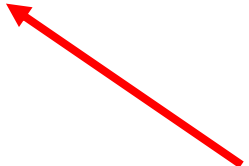
← Note: variable names can be omitted

C++ templates - TMP

The generic implementation:

```
template<typename x> struct sq  
{  
    using result = decltype (x() * x ());  
};
```

This creates an
instance of type `x`



C++ templates - TMP

But wait...

Why this needs to be a class template?

C++ templates - TMP

```
template< typename X >  
auto sq(X x){ return x*x; }
```

This function now works for both values, and both template level stuff, like `Int<4>`:

```
some_class< decltype( sq( Int<4>() ) ) > c;
```

C++ templates - TMP

```
template< typename X >  
auto sq(X x){ return x*x; }
```

After `decltype`, we get back to type level, and can use the type of the result in templates, utilize specialization, etc.

Use all our operators and generic functions inside the `decltype`

We first create a value of the type `Int`

```
some_class< decltype( sq( Int<4>() ) ) > c;
```

C++ templates - TMP

```
template< typename X >  
auto sq(X x){ return x*x; }
```

```
some_class< decltype( sq( Int<4>() ) ) > c;
```

It will be equivalent to



```
some_class< Int<16> > c;
```

C++ templates - TMP

More metaprogramming:

Type level lists (or arrays, as you like)

C++ templates - TMP

Type level lists:

```
template<typename... elems> struct List;
```

```
using xs = List<Int<1>, Int<2>, Int<3>>>;
```

C++ templates - TMP

Type level lists:

Since C++11 [variadic templates](#) are allowed, that can represent 0 or more template arguments of any type!



```
template<typename... elems> struct List;
```

```
using xs = List<Int<1>, Int<2>, Int<3>>>;
```

C++ templates - TMP

Ok, but how we get elements out?

C++ templates - TMP

Ok, but how we get elements out?

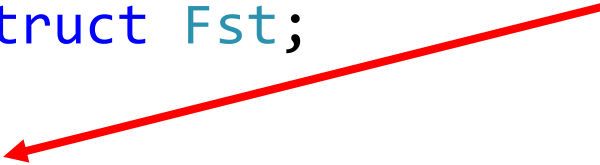
First element:

```
template<typename... elems> struct List;
```

Usual trick: break the pack into first and rest!

```
template<typename L> struct Fst;
```

```
template<typename A0, typename... As>  
struct Fst<List<A0, As...>>{ using result = A0; };
```



The list will be pattern matched against `A0` and `As...`

```
using x0 = typename Fst<List<Int<1>, Int<2>, Int<3>>>::result;
```



Will be `Int<1>`

C++ templates - TMP

Ok, but how we get the other elements out?

Recursion!

C++ templates - TMP

```
//Helper struct
template<typename Icurrent, typename Igoal, typename L> struct Nth_impl;

template<typename Icurrent, typename Igoal, typename A, typename... As>
struct Nth_impl<Icurrent, Igoal, List<A, As...>> //The induction step
{
    using result = typename Nth_impl<Int<Icurrent::result+1>,
                                   Igoal, List<As...>>::result;
};

template<typename I, typename A, typename... As> //Closing step of recursion
struct Nth_impl<I, I, List<A, As...>> { using result = A; };

template<typename L, typename I> //End-user alias
using Nth = typename Nth_impl<Int<0>, I, L>::result;
```

C++ templates - TMP

```
//Helper struct
template<typename Icurrent, typename Igoal, typename L> struct Nth_impl;

template<typename Icurrent, typename Igoal, typename A, typename... As>
struct Nth_impl<Icurrent, Igoal, List<A, As...>> //The induction step
{
    using result = typename Nth_impl<Int<Icurrent::result+1>,
                                   Igoal, List<As...>>::result;
};

template<typename I, typename A, typename... As> //Closing step of recursion
struct Nth_impl<I, I, List<A, As...>> { using result = A; };
```

The recursion ends,
when `Icurrent == Igoal`

This is just the same as `Fst` we just wrote!


C++ templates - TMP

Lets write this out on an example:

```
L = List<Int<1>, Int<2>, Int<3>>, Igoal = Int<2>
```


Step 1:

```
struct Nth_impl<Int<0>, Int<2>, List<Int<1>, Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>> >::result;
};
```



Step 2:

```
struct Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<2>, Int<2>, List<Int<3>> >::result;
};
```



This instantiates step 2.:

The recursion ends,
because Icurrent == Igoal

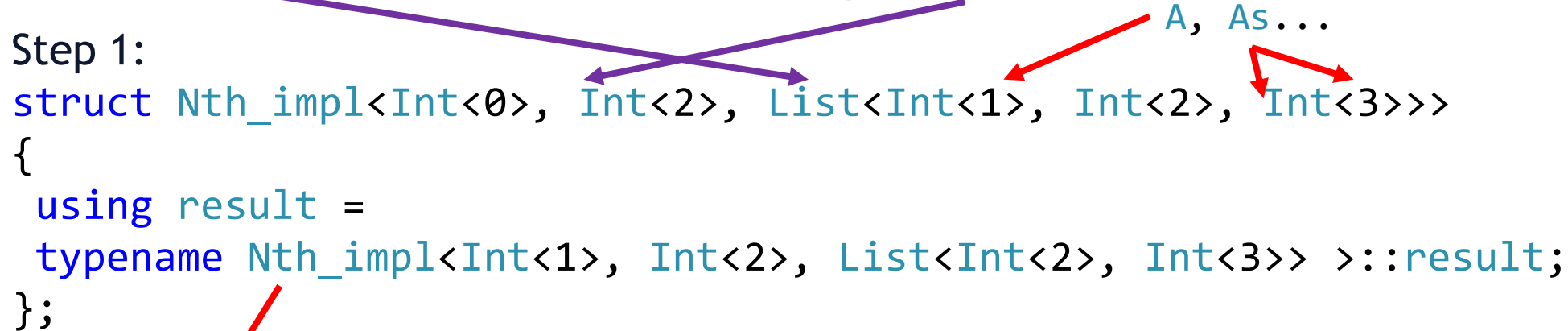
C++ templates - TMP

Lets write this out on an example:

```
L = List<Int<1>, Int<2>, Int<3>>, Igoal = Int<2>
```


Step 1:

```
struct Nth_impl<Int<0>, Int<2>, List<Int<1>, Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>> >::result;
};
```



Step 2:

```
struct Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<2>, Int<2>, List<Int<3>> >::result;
};
```



This instantiates step 2.:

The recursion ends,
because Icurrent == Igoal

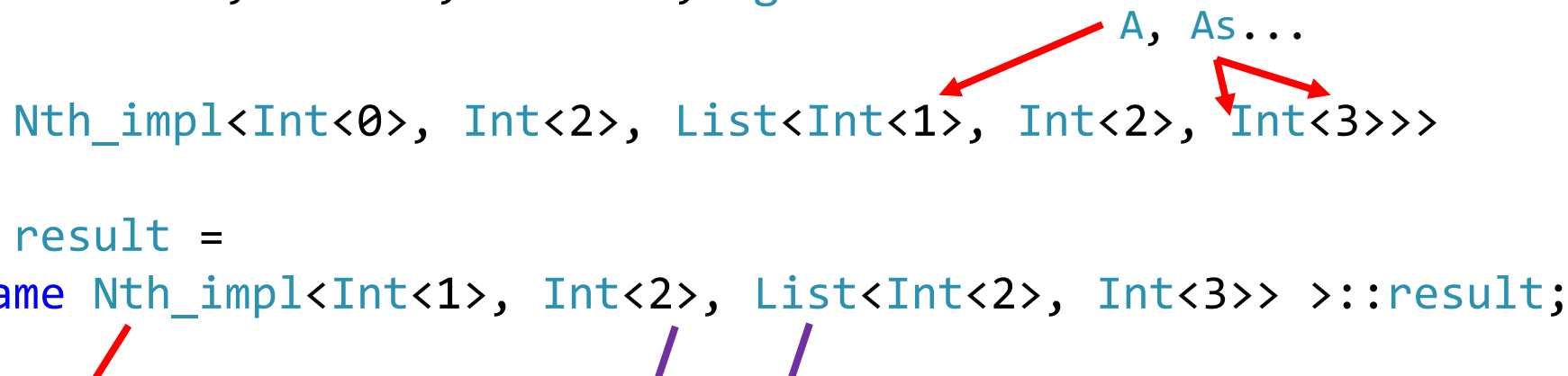
C++ templates - TMP

Lets write this out on an example:

```
L = List<Int<1>, Int<2>, Int<3>>, Igoal = Int<2>
```

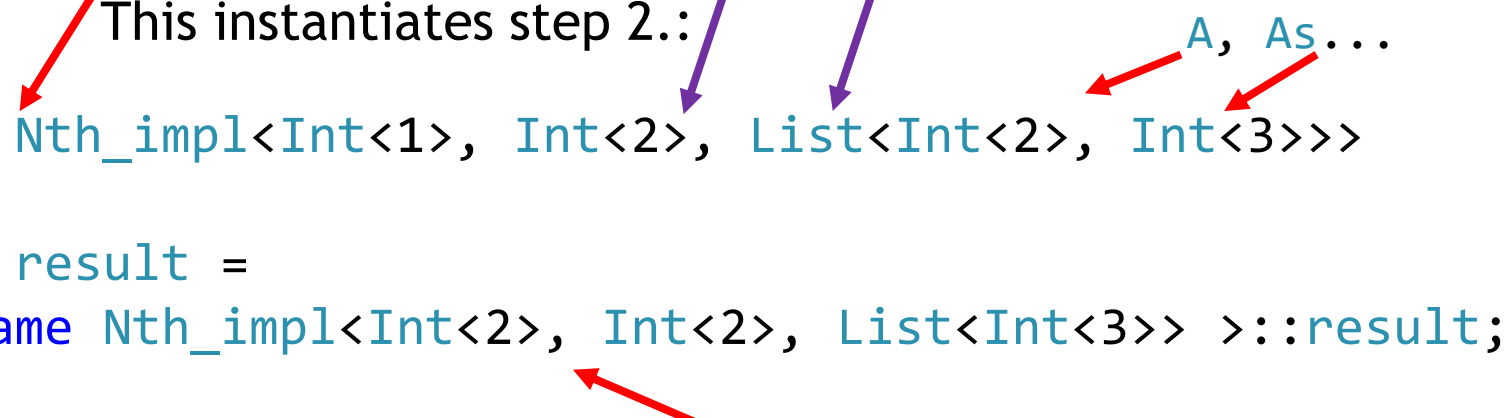
Step 1:

```
struct Nth_impl<Int<0>, Int<2>, List<Int<1>, Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>> >::result;
};
```



Step 2:

```
struct Nth_impl<Int<1>, Int<2>, List<Int<2>, Int<3>>>
{
    using result =
    typename Nth_impl<Int<2>, Int<2>, List<Int<3>> >::result;
};
```



This instantiates step 2.:

The recursion ends,
because `Icurrent == Igoal`

C++ templates - TMP

Ok, but how we get the other elements out?

```
using xn = typename Nth< List<Int<1>, Int<2>, Int<3>>, Int<n> >::result;
```

We can again hide it behind an operator:

```
template<typename... L> struct List  
{  
    template<int n>  
    auto operator[] ( Int<n> )  
    { return Nth<List<L...>, Int<n> >(); }  
};
```


C++ templates - TMP

Okay, we have values, functions, lists, what can we do with them?

C++ templates - TMP

Typical applications of template-metaprogramming:

- Dealing with structures and functions operating on variadic (but compile time known) number of arguments:

implementing [std::tuple](#)

checking type traits on collections of types
and other type level algorithms

creating a type-safe version of
`printf("value is %i", x)`

C++ templates - TMP

Typical applications of template-metaprogramming:

- Explicit inlined repetition of parametric tasks

you might use a loop, but you either want to be sure, that it will be unrolled, or

you need the result at compile time!

(e.g. to use it in a template, c.f. integer sequences, partitions, combinatorics, precalculated exact rational expressions...)

C++ templates - TMP

Typical applications of template-metaprogramming:

- Concrete examples:

Runge-Kutta steppers ← See the advanced samples
for arbitrary stages

Finite-Difference stencils
for multiple dimensions, arbitrary order

C++ templates - TMP

Another major application:

Expression templates!

C++ templates - TMP

Idea:

What if we want to do something with the *structure* of a computation?

C++ templates - TMP

Idea:

What if we want to do something with the *structure* of a computation?

Lazy evaluation

C++ templates - TMP

Expression templates - simple example

```
template<typename T>
struct Variable
{
    T val;
    auto operator()() const { return val; }
};
```

```
template<typename T>
auto var( T const t ){ return Variable<T>{t}; }
```

```
template<typename T>
bool is_expression( Variable<T> ){ return true; }
```


C++ templates - TMP

Expression templates - simple example

```
template<typename T>
struct Variable
{
    T val;
    auto operator()() const { return val; }
};
```

Simple struct to hold any type of value,
and provide it when () is called

```
template<typename T>
auto var( T const t ){ return Variable<T>{t}; }
```

Helper function to create a
`Variable<T>` (infers the template type)

```
template<typename T>
bool is_expression( Variable<T> ){ return true; }
```

Helper function that says, `Variable<T>`
is an expression (see later)

C++ templates - TMP

Expression templates - simple example

```
template<typename Op, typename Left, typename Right> struct Expression
{
    Op op; Left left; Right right;
    auto operator()() const { return op(left(), right()); }
};
```

```
template<typename Op, typename Left, typename Right>
auto expr( Op const& op, Left const& l, Right const& r )
{ return Expression<Op, Left, Right>{op, l, r}; }
```

```
template<typename... T> bool is_expression( Expression<T...> ){ return true; }
```

C++ templates - TMP

Expression templates - simple example

A struct to hold a binary function, and two (callable) types, when () is called, op is evaluated on the results of left() and right()

```
template<typename Op, typename Left, typename Right> struct Expression
{
    Op op; Left left; Right right;
    auto operator()() const { return op(left(), right()); }
};
```

Helper function to create an `Expression` (infers the template types)

```
template<typename Op, typename Left, typename Right>
auto expr( Op const& op, Left const& l, Right const& r )
{ return Expression<Op, Left, Right>{op, l, r}; }
```

Helper function that says, `Expression<T...>` is an expression (see later)

```
template<typename... T> bool is_expression( Expression<T...> ){ return true; }
```

C++ templates - TMP

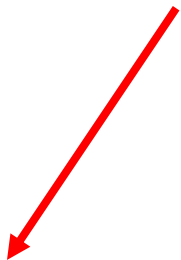
Expression templates - simple example

And now an addition operator:

here we use the SFINAE with the `is_expression` helper functions to restrict this operator only to our types defined above.

```
template<typename Left,  
        typename Right,  
        typename Rq1 = decltype(is_expression(std::declval<Left>())),  
        typename Rq2 = decltype(is_expression(std::declval<Right>()))>  
auto operator+( Left const& l, Right const& r )  
{  
    return expr([](auto x, auto y){ return x+y; }, l, r);  
}
```

`std::declval` is a helper function to return a `T&&` to any type. Only to be used inside a `decltype`



C++ templates - TMP

Expression templates - simple example

And now a multiplication:

```
template<typename Left,  
        typename Right,  
        typename Rq1 = decltype(is_expression(std::declval<Left>())),  
        typename Rq2 = decltype(is_expression(std::declval<Right>()))>  
auto operator*( Left const& l, Right const& r )  
{  
    return expr([](auto x, auto y){ return x*y; }, l, r);  
}
```

C++ templates - TMP

Expression templates - simple example

And NOW if we write:

```
auto c = (var(2.0) + var(4.0)) * var(5.0);  
std::cout << "type of c is: " << typeid(c).name() << " value is: "  
          << c() << std::endl;
```

The output is (reformatted for readability):

```
type of c is:  
struct Expression< class <lambda_9a68...>,  
                  struct Expression< class <lambda_29b5...>,  
                  struct Variable<double>,  
                  struct Variable<double> >,  
                  struct Variable<double> >  
value is: 30.000000000000000000
```

This is our multiplication lambda inside operator*

This is our addition lambda inside operator+

C++ templates - TMP

Why is it good to have the expression trees?

We can do whatever we want *before* evaluation.

Example: see the simple symbolic differentiation example,
it includes the printer of the expressions to the console

Symbolic optimizations on such trees are very common.

C++ templates - TMP

Why is it good to have the expression trees?

A [similar method](#) that is using the [CRTP pattern](#) is widely used for handling vector and matrix expressions.

It builds a similar tree, but it uses `operator[]` as the common interface function to extract values

while avoiding the allocations of temporaries!!!

C++ templates - TMP

While some of these metaprogramming tricks are known since 20 years, they did not make into the standard library.

However, the [boost](#) library collection heavy uses such techniques.

In STL 2 such expression template based constructs will be used more often.